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LETTER TO THE EDITOR

Dynamic scaling of the interface in a diffusive front

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Abstract. A model is introduced to simulate irreversible wetting in a two-dimensional lattice system in which a fixed number of carriers, each diffusing from the source, wet the dry sites on their trail. We find that the propagation of the front of the wet phase is diffusive. The interface width is found to increase as a power of the average height with the exponent $\beta \simeq 0.74$. In a system of finite size L the width saturates to a constant value in the long time limit. The saturated interface width scales as L^α with $\alpha \simeq 1$.

Recently there has been considerable interest in understanding the dynamics of non-equilibrium interface growth phenomena [1] in the context of a variety of models, analytical theories and experiments. A classic example is the dynamics of the motion of the interface between a wetting fluid, like water, and a dry surface which is being wetted irreversibly as the fluid front moves forward. In this letter we present a simple model to simulate this sort of irreversible wetting in a lattice system and we study the dynamics of the interface.

Much of the recent interest in the study of the dynamics of interfaces has been based on the fact that surface fluctuations exhibit scaling behaviour in both time and space. In particular, assuming an initially flat interface, the scaling of the width of an interface is expected to be of the form [2]

$$w(L, \bar{h}) = L^\alpha f(\bar{h}/L^z)$$

where $w(L, \bar{h})$ is the interface width on length scale L and $\bar{h}$ is the average height of the interface. The dynamic exponent $z = \alpha/\beta$, and the scaling function $f(x) \sim x^\beta$ for $x \ll 1$ and $f(x) \to$ constant for $x \gg 1$. Consequently, most of the recent investigations of the scaling properties of surfaces and interfaces are focused on the determination of the exponents α and β, and on the analysis of the dynamical scaling in surface and interface growth. A variety of the models [1] (including aggregation, deposition and Eden models) in two dimensions seem to give the values $\alpha = \frac{1}{2}$ and $\beta = \frac{1}{3}$ in agreement with the analytical results of Kardar *et al* [3] (KPZ) based on a nonlinear Langevin-type equation. However, more recently it has been shown that in experiments on two-fluid flow in porous media [4, 5] and in a number of models [6–10], the exponents are anomalous and do not agree with the KPZ result. In most of the previous studies, either the growth of the height with time is not analysed in detail or the mean front is found to grow linearly with time except in a few studies [11] where the front is found to evolve non-diffusively. Here we introduce and study the dynamics of the interface in a model consisting of a fixed number of carriers, each diffusing from the source, which

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wet the dry sites on their trail on a two-dimensional lattice. The interface separating the wetted region from the dry region resembles the front in the flow of a wetting fluid into a porous media.

We consider a two-dimensional lattice of width L and infinite length, which is initially considered to be dry. A reservoir source of wetting fluid is connected with all the sites at the shorter side of the lattice. Wetting particles (the carriers) are then placed at each lattice site connected to the reservoir of the wetting fluid. In this model we assume that there is a hard-core interaction between the particles so that a lattice site cannot be occupied by more than one particle at any given time. Furthermore, unlike gradient-induced percolation [12], we have used a fixed number, L, of particles in our simulations. The wetting particles perform random walks on the lattice and *all* the lattice sites occupied or visited by particles become permanently wet. The particles spread the fluid into the dry lattice as they execute their stochastic motion with the following rule. We randomly select a particle at a site i and one of its neighbouring sites j. If the site j is empty, then the particle is moved from site i to site j; the site j becomes permanently wet, if it was dry before. If randomly selected site j is occupied by another carrier, then an attempt to move the particle from site i to site j fails and the particle remains at site i. An attempt to move each particle once is defined as one Monte Carlo step (MCS) time. We use a periodic boundary condition along the transverse (y) direction, and an open (reflecting) boundary condition at the source. In the simulations the length along the infinite direction (x) is chosen such that the particles never reach the opposite side of the source.

We note that our choice of exactly L walkers and the hard-core interaction are not important restrictions of the model. We used L particles for simulational convenience, but any finite value of L can be chosen by feeding in particles continuously at the source. Similarly, we introduced the hard-core repulsion in order to simulate real particles, but we expect that in the long time limit this exclusion principle might be irrelevant. We plan to extend this work by carrying out further studies with different number of walkers and with different types of interacting and non-interacting diffusers.

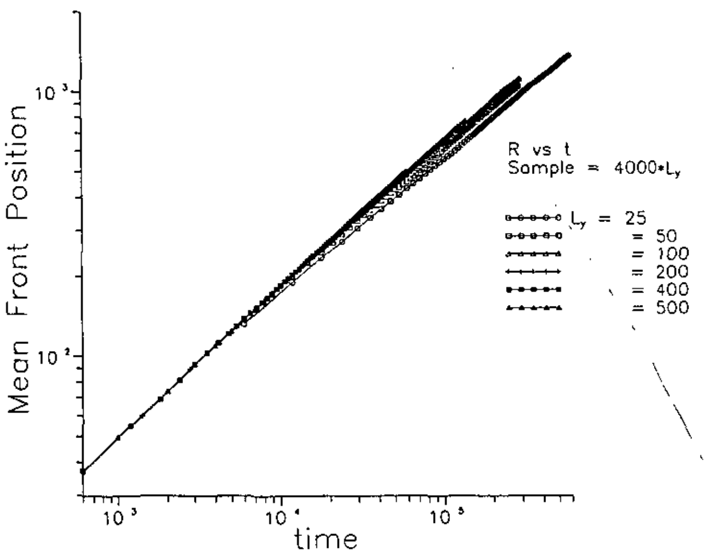

Figure 1. Mean front position $\bar{h}$ versus time on a log–log plot. Samples of width $L = 500$, 400, 200, 100, 50, 25 are used with up to 500 independent runs.

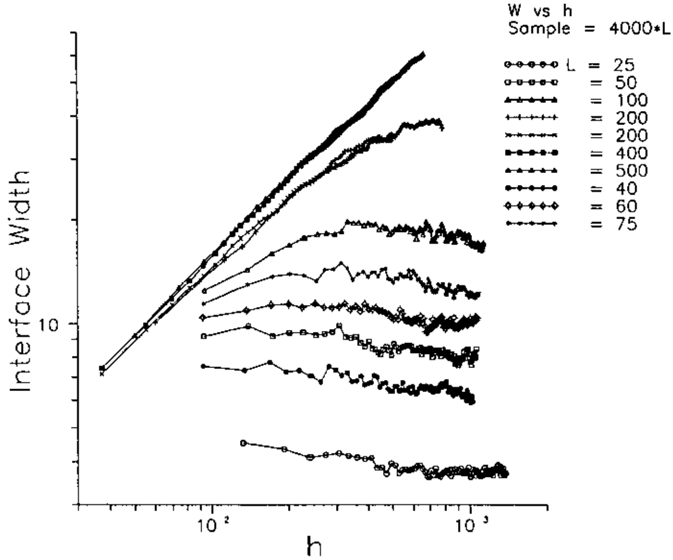

Figure 2. Width of the interface w versus $\bar{h}$ on a log-log plot for length $L=25$, 40, 50, 60, 75, 100, 200, 400 and 500 with the same statistics as in figure 1. The slope of the linear region (before the saturation) is about 0.74.

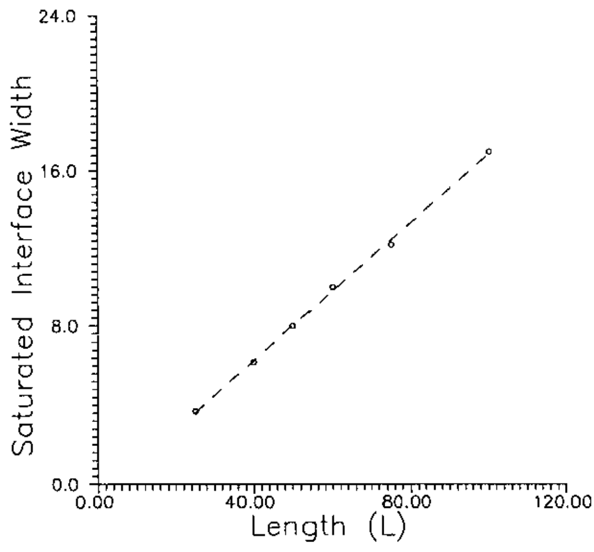

Figure 3. Saturated interface width W versus size L on a log-log plot. The slope is around 1.

The fluid front moves into the dry lattice and an interface develops between the wet and the dry phase of the lattice as the particles execute their stochastic hopping. We have studied in detail both the motion of the wet front and the growth of the interface width as a function of time for several sample sizes each with a number of independent runs. We define the height $h(i)$ on the front as the last wet site in row i. This amounts to ignoring the overhangs which are not assumed to play an important role in wetting phenomena and in this model. The average position of the front $\bar{h}$ is defined by $\bar{h}=\Sigma_{i=1}^{L} h(i)/L$. A typical plot of the mean front position $\bar{h}$ versus time t is shown in figure 1; the slope of the log–log plot leads to an exponent $k=\frac{1}{2}$ in $\bar{h}\sim t^{k}$. This indicates that the front moves diffusively.

The width of the interface, $w(L, t)$, is defined as the standard deviation of the heights $h(i)$ and is computed from the relation $w(L, t)^2=\Sigma_{i=1}^{L}(h(i)-\bar{h})^2/L$. The plot of the interface width w versus $\bar{h}$ is presented in figure 2 on a log–log plot. We find that $w\sim\bar{h}^{\beta}$ with the exponent $\beta=0.74\pm0.04$ (see the data with large samples, $L=400$ and 500). We have also analysed the size dependence of the interface width with sizes ranging from $L=25$ to 500. However, we have achieved the saturation in width only up to $L=200$; the larger samples require much more computer time for a reliable estimate. A plot of the saturated interface width versus L is shown in figure 3. In the asymptotic regime, $w\sim L^{\alpha}$ and we find $\alpha\simeq 1$. Our estimates of α and β indicate that the KPZ scaling relation $\alpha+\alpha/\beta=2$ is not fully consistent with the data, but this possibility cannot be completely ruled out due to the large error in the values of the exponents. However, we are not aware of a compelling argument that would suggest that our model should be related to the KPZ model.

Larralde *et al* [13] have recently presented a similar model and studied the growth of the colony invaded by N diffusers starting from a single site. In particular they analytically obtained the number of distinct visited sites in time t by N diffusers released from a single (spherically symmetric) point source. Besides the difference in the geometries of the source from our study (pointlike sources versus a line of wetting fluid), they also did not analyse the growth of the interface roughness with time and length. It might be interesting to further study the dynamics of the rough surface with a spherically symmetric (pointlike) fluid source.

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